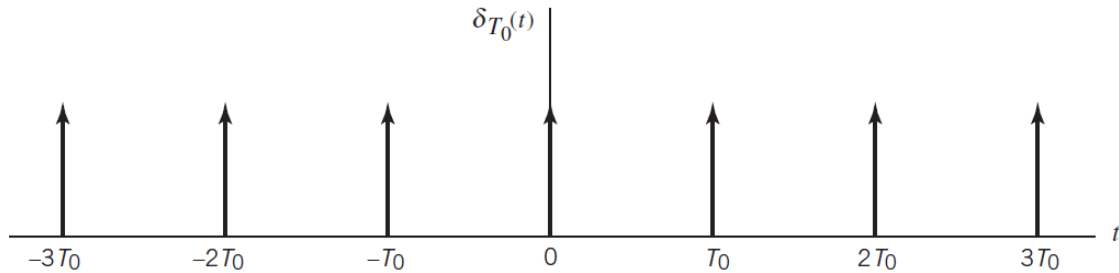
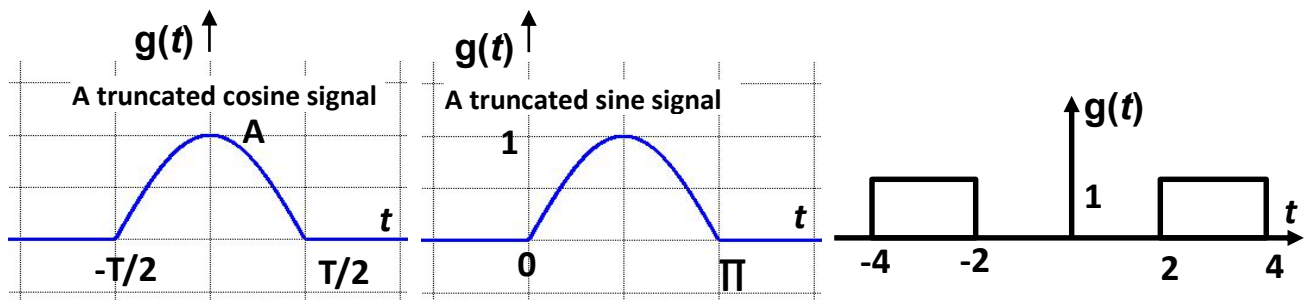


- 1- Find the trigonometric and exponential Fourier series of the periodic impulse train (Dirac comb function) shown in figure



- 2- For each of the following signals: (a) express $g(t)$ as a function of time, and (b) find its Fourier transform



- 3- Sketch $G(W) = \text{rect}((W-4)/2)$, then find and sketch the inverse Fourier transform of $G(W)$.
- 4- Prove the following:
- The dilation "Scaling" property of the Fourier transform
 - The duality property of the Fourier transform
 - The time-shifting property
 - Use the duality property to prove the frequency-shifting property
- 5- Assuming that f_c is larger than $(1/T)$, determine the Fourier transform of the radio frequency RF pulse: $g(t) = A \text{rect}\left(\frac{t}{T}\right) \cos(2\pi f_c t)$
- 6- Using differentiation property, find and sketch the Fourier transform of the triangle pulse

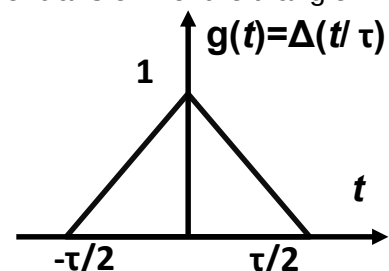


Table of Fourier transform

No.	$x(t)$	$X(\omega)$
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega} \quad a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega} \quad a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2} \quad a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2} \quad a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}} \quad a > 0$
6	$\delta(t)$	1
7	1	$2\pi\delta(\omega)$
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
12	$\text{sgn } t$	$\frac{2}{j\omega}$
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$
15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2} \quad a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2} \quad a > 0$
17	$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$
18	$\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}\left(\frac{\omega}{2W}\right)$
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$
20	$\frac{W}{2\pi} \text{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) \quad \omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$

Properties of Fourier transform

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^t x(u) du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Fourier transform

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t} dt$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega)e^{j\omega t} d\omega$$

Fourier series

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int_{T_0} g(t)e^{-jn\omega_0 t} dt$$